EVALUATION OF NON-COOPERATIVE GAMES AS A TOOL FOR ANALYZING POLITICAL AND POLICY BEHAVIORS**

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Abstract: This study evaluates the limitations and relevance of non-cooperative games as a tool for analyzing political and policy behavior. It aims to answer the questions, "for what situations are such non- cooperative games appropriate, and for what do they fail to predict actual behavior?" To answer these questions, we are goint ot look into the assumptions underlying the existence of equilibria developed by this non-cooperative game model. This model uses two types of situations: extensive form and normal form. Each of them is evaluated in terms of its underlying assumptions and their relevance to real behaviors in the public sector.

The several theorems concerning non-cooperative game are introduced and evaluated in terms of its meaning and relevance to the real public arena. In particular, we see that the zero sum assumption is a restriction that precludes modeling many other situations central to politics and public administration, and that the prisoner dilemma models important processes, including collective action problem.

INTRODUCTION

The theory of games is essentially nothing but a mathematical theory of conflict situations. Game theory began as applied mathematics, but it is now central to the way we think about business and economics. The object of game theory is to analyze and elaborate what constitutes rational behavior of each of the opponents in the course of a conflicting situation (Myerson, 1991: 1~2). Every conflicting situation, taken as it is from a practical field, is very complex with numerous attendant factors, making it difficult to analyze. To mathematically analyze such a situation, it is necessary to avoid secondary attendant factors and to build a simple, formalized model of the situations. Such a model is called a game.

In general, games are divided into two categories: non-cooperative games and cooperative games (Nash, 1996). In its traditional interpretation, the theory of non-cooperative games assumes that while fates are interdependent, people cannot coordinate their choices. On the other hand, cooperative game is a game in which the players have complete freedom of preplay communication to make joint binding agreements. In a non-cooperative game, absolutely no preplay communication is permitted between the players (Luce and Raiffa, 1957: 89). Politically, people can manipulate the outcomes of institutions by strategically revealing their preferences. Those institutions may range from formal voting mechanism to informal social processes. The consequences of one's actions, however, should depend on what others do, and, thus, the preference people reveal should depend on what preferences they think others will reveal. It is clear that this interdependence is fundamental to politics (Ordeshook, 1986: 97).

As noted above, while the object of game theory is to analyze and elaborate what constitutes rational behavior in conflicting situations, the properties of dominant rational behavior or equilibria vary,

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depending on the situation being analyzed and the assumptions that are used to model the situation. Hence, any choice between a non-cooperative and cooperative formulation of a situation may be debatable. According to Ordeshook (1986: 99), a non-cooperative game is a situation in which people must choose without the opportunity for explicitly coordinating their actions. In other words, if no two persons can establish a contractual arrangement whereby their decisions are unambiguously linked, then a game is non-cooperative.

In this study, our discussion will be focused specifically on the non-cooperative game. In doing so, several non-cooperative game theorems are introduced and evaluated in terms of its meaning and relevance to the real political arena. In particular, we will concern the assumptions underlying the existence of equilibria developed by the non-cooperative game model.

THE LIMITATIONS OF NON-COOPERATIVE GAMES FOR ANALYZING POLITICAL BEHAVIOR

In order to discuss the limitations of non-cooperative game, there may be at least two questions to pose. First, how realistic are non-cooperative situations? Second, what are the assumptions of theorems concerning equilibria of non-cooperative games? The first question is closely related to the second, and these two questions are concerned with such topics as "for what situations are such games appropriate, and for what do they fail to predict actual behavior?" Thus focus will be on the latter questions.

Game theory uses three kinds of descriptions to model situations: (1) the extensive, or game-tree form; (2) the normal, or strategic form; and (3) the characteristic-function form. The third form concerns cooperative games. Thus we are concerned only with the extensive and normal forms. We will start with the extensive form.

Extensive Form

The extensive form is the basic description of a game. It is a tree diagram with nodes, branches, an initial node, and information sets.[1] The extensive form (Gardner, 1995: 10~11) is an elementary description of the situation that we are modeling, and it contains the following information: whose choice (move) is it at any particular point in time? What alternative actions are available to each person at any particular move? What does each player know about other person's prior choices? What are the alternative states of nature and their likelihood? Finally, what are each person's preferences or utilities over the outcome? With these in mind, there may be several things to be commented (Ordeshook, 1986: 100~106).

First, every extensive form must be necessarily incomplete because the only truly complete form is one that models all of future history. However, this does not seem critical. Instead, we should recognize that an extensive form of game provides the foundation to develop the basic principles of game theory. It should be noted that with this extensive form, we cannot describe games in which people enjoy an infinite number of strategies, such as position on issues, money to be allocated in the pursuit of some public activity, and the timing of a particular act. A strategy is a complete plan of play for a game.[2]

Second, we presently assume that a person's information set summarizes whatever uncertainty a

1) The extensive form is a tree diagram, so called because it looks like a tree if you are at the starting point, facing right. Points in the tree are called nodes. Every game begins with an initial node. An information set shows which player has to move and what the player knows when making the move. The lines coming out from a node are called branches.

2) Strictly speaking, a strategy in the game theory is a rule for a person that, on any particular move, selects just one act from each informational set.

person has about a situation. Thus, all persons know the properties of a game's extensive form, including the utility function of other persons (at least up to some probability distribution), their alternatives, and the characteristics of their information sets. This assumption is unrealistic in many applications, such as when some legislators do not know the preferences of other legislators or of their constituents.

Normal Form

The second form of descriptions is normal form. Unlike extensive form, one of its advantages is that it can describe infinite games. The most important concept of equilibrium based on this normal form is one of pure-strategy equilibria. A pure strategy means that a person always selects the same sequence of acts on any play of a game if we hold constant the choices of the other persons and the prevailing states of nature. Now, let's take a closer look at the concept of Nash equilibrium. Each person, i, selects a strategy, s_i, from the set of strategies, S_i, available to i before knowing the choice that any other person makes. The utility function on the joint actions that all n persons take, $u_i(s_1, s_2, \Lambda, s_i, \Lambda, s^*_n)$, gives each player's preferences. Suppose there is a particular vector of strategies for the players of an n-person non-cooperative game $s^*=(s^*_1, s^*_2, \Lambda, s^*_n)$, such that no player has an incentive to change his strategy in this vector if he believes that everyone else will choose his corresponding strategy in the vector. Put differently, suppose that s^*_i is a best-response strategy to s^*_{-i} for all i. We call such a vector an equilibrium point of the non-cooperative game, otherwise known as the Nash equilibrium.3) Thus, s^* if is an equilibrium, and if each person i expects the others to adopt s^*_{-i}, then i can do no better by choosing some strategy other than s^*_i. In fact, i may even do worse.

There are some basic theorems in the theory of non-cooperative games concerning the existence of pure-strategy equilibria. That is, each theorem guarantees that any game satisfying certain conditions has at least one pure-strategy equilibrium, n-tuple, of strategies. One of such theorems is as follows (Ordeshook, 1986: 120):

(Theorem 1) Every finite non-cooperative game of perfect information has at least one pure strategy equilibrium n-tuple.

A game of perfect information is a game in which every information set in its extensive form contains just one choice point (Myerson, 1991: 44～45). Therefore, each player makes each choice with the full knowledge of earlier choices made in the exact path in the game tree. Even though we can expect that roll call voting satisfies the perfect information condition, it will clearly show that the assumptions of perfect information and determinate choice seem to limit the application of the theorems to real world situations. And there is a theorem based on a concave game in normal form (Ordeshook, 1986: 128):

(Theorem 2) Every concave game in normal form has at least one equilibrium in pure strategies. A concave game in normal form is a non-cooperative game such that (1) the strategy set of each player i, S_i, is a convex closed and bounded subset of m-dimensional euclidean space, and (2) for every player i and for every strategy n-tuple $s=(s_1, s_2, \Lambda, s_n)$, the utility function $u_i(s)$ is both continuous in s and concave in the ith player's strategy, s_i, holding the other players strategies, s_{-i}, fixed.

This game's conditions satisfy every condition of a fixed point theorem, which is used to prove the

3) $\mathbf{s}_{-i}=(s_1, \Lambda, s_{i-1}, s_{i+1}, \Lambda, s_n)$ identifies a choice of strategy for all persons except i. Using this notation, we can now express the choice of strategies by all persons, $\mathbf{s}=(s_1, s_2, \Lambda, s_n)$, as $\mathbf{s}=(s_i, \mathbf{s}_{-i})$. Then, person i's strategy s_i^* is a best response against a specific choice of strategies by the remaining players in the game, $\mathbf{s}_{-i}$, if and only if $u_i(s_i^*, \mathbf{s}_{-i}) \geq u_i(s_i, \mathbf{s}_{-i})$, for every s_i in $\mathbf{S}_i$.

above theorem. But there may be some problems in the assumptions of convexity and compactness of strategy space (Schwartz, 1986). Although political analysts have considerable latitude in identifying the feasible set or strategy space in any given case, the feasible sets commonly identified are finite, and they are constrained in innumerable ways. Often a feasible set is comprised of the candidates for some office, often the motions voted on in some legislature or committee. In either case it is finite. One might try to construe the feasible set in an election as comprising all possible platforms a candidate might conceivably adopt. Although infinite, such a set cannot be convex if platforms are formulated in English; there are only many English sentences. One might try to construe a legislature's feasible set as comprising all possible motions or motion bundles, not just those actually moved. But again, there are only many of those. And besides budgets and technology, there are constitutional and procedural hurdles that a possible motion must clear to qualify as feasible--to be allowed on the agenda.

There is also a theorem based on mixed extension (Ordeshook, 1986: 135):

(Theorem 3) Every mixed extension of a finite, n-person, non-cooperative game has at least one equilibrium n-tuple in either pure or mixed strategies.

This is actually a corollary of Theorem 2, since every finite n-person, non-cooperative game can be transformed into a concave game through mixed extension. Therefore, the limitations given to Theorem 2 may be true in this theorem. In particular, it is not transparent how much this theorem could be relevant to politics.

At this point, we need to note a theorem based on zero sum game, which is called the saddle point theorem(Ordeshook, 1986: 154):

(Theorem 4) For two-person zero sum games, if A and B are convex, closed, and bounded, and if $u_i(a, b)$ is concave in a and convex in b, as well as continuous in a and b for all $a \in A$ and $b \in B$, then the game has an equilibrium in pure strategies.

If a game has zero sum, then each person's utility function is convex in his opponent's strategies. Here, if we substitute the words "strictly concave" and "strictly convex" for "concave" and "convex," respectively, then the equilibrium is much stronger than that of Theorem 2 or 3. Thus its application is almost limited to zero sum games. In addition, the concept of perfect equilibria was developed to avoid the problem that arises when moving from an extensive form to a normal form; we may lose information about a game's strategic character. Since this concept is based on an extensive form, the limitations given to that form may be true in this concept.

The Relevance of Non-cooperative Games

Then, for what situations are such non-cooperative games appropriate? It is obvious that non-cooperative games are appropriate for non-cooperative situations. An extensive form will be appropriate for simple voting games and roll call voting, especially if roll call voting satisfies a perfect information conditio n.[4] Thus we can expect that (Theorem 1) will be true in that situation. As proved by Ordeshook (1986: 155~157), (Theorem 4) the saddle point

4) We can relax this assumption by introducing 'nature' concept. For example, it is possible to incorporate the possibility that each legislator is unsure about how constituents will react to a vote for a salary increase. Constituents, as a whole, may regard the issue of legislative salaries as a critical determinant of their vote in the next election, or they may be insensitive or unaware of the issue owing to little press coverage. If they are insensitive to the issue, they neither reward nor punish their legislators for how they vote. In this case, we can incorporate this possibility by assigning nature a final move in the game, in which nature "choose" between a sensitive and an insensitive electorate with each probability.

theorem is appropriate for the game of resource allocations in American elections. Consider a game with two candidates who compete by allocating resources to fifty states. In this game, the strategy space of each candidate is convex and compact, and each candidate's defined utility function is continuous to his strategies and convex to opponent's strategies. Thus, as predicted by Theorem 4, in equilibrium both candidates allocate their resources in proportion to each state's electoral vote weight.

But we should not interpret this analysis as a general conclusion about campaigning under the electoral college in American politics. First, it ignores the dynamic aspects of campaigning and that candidates often adjust their campaign tactics as the election approaches. While the analysis supposes that candidates must make a single allocation decision that must serve them for all time, the preceding normal form perhaps oversimplifies an election's true extensive form by ignoring the dynamic nature of campaigns, which permits continual adjustment of allocations. Second, utility function defined in that game imposes a special assumption about the relationship between resources and the probability of carrying a state. While that assumption guarantees the concavity of payoff functions and the existence of an equilibrium, it is not necessarily the most plausible possibility.

Nevertheless, this example illustrates how theorizing about the electoral college might proceed. This model shows that there is no large- or small-bias insofar as resources are concerned, and refinements might yield conclusions which we regard as satisfactory for policy prescription. This example further illustrates a symmetric two-person game in which both candidates hold identical resources, and no candidate enjoys an advantage in any state.[5] Hence, we could conclude directly that for at least one equilibrium, the candidates allocate their resources in an identical pattern. And since payoffs are strictly concave, we know that this is the unique equilibrium.

To illustrate these ideas about symmetric games, another example was given by Ordeshook (1986: 158～166). It is concerned with the question of the equivalence of alternative candidate objective in elections: maximizing expected plurality or maximizing probability of winning. At this point, game theory provides a definition: two objectives are equivalent if they yield identical equilibria and, thus, identical prediction about outcomes. In this example, it is concluded that for symmetric games and symmetric representations of uncertainty, the two objectives are equivalent. Because objectives, such as maximizing probability of winning or maximizing plurality, yield zero sum payoffs to candidates, the study of two-candidate elections is a principal application of two-person zero sum game theory to politics.

In fact, the zero sum application is a restriction that precludes modeling many other situations that are central to politics. Ordeshook (1986: 203～242) explores more general classes of non-zero, non-cooperative games by focusing on a particular

5) A game is a symmetric game when it looks the same to each player. "Looks the same" boils down to two major considerations. First, each player i has the same set of strategy S_i; one player does not have more strategies than, or different strategies from, another. Second, every pair of players i and j have the same utility function in the sense that, given the strategies of all the other players, interchanging the strategies of players interchanges their payoffs. Therefore, two-person game is symmetric if the strategy sets of both players are identical (if $A=B$), and if, when the players exchange strategies, they also exchange payoffs. That is, $u_1(a, b)=u_2(b, a)$ for all $a \in A$ and $b \in B$. Symmetric games are important for two reasons. First, they provide good approximations to complicated games when the players are not very different, and they are easier to solve. Second, symmetric games inspire a sufficient condition for a solution. Since a symmetric game looks the same to every player, there is no reason to believe that one player has an advantage over another. All players have equal opportunity. The solution to a symmetric game should reflect this equality of opportunity.

game: the prisoner's dilemma. The prisoner dilemma models important political processes, including collective action problem.[6] Many of the publicly stated justifications for governmental incursions into our lives and for the formulation of specific public policies are arguments for avoiding real or imagined dilemmas. The prisoner's dilemma also provides a simple hypothesis as to why governments overexpand and render inefficient outcomes. For example, as Aranson and Ordeshook (1977 & 1985) observed in their formulation of the causes and consequences of government failure, the goods and services that governments provide necessarily have a dual character. Although defense, education, and the like may be public and the costs of providing such goods through a general tax-revenue system may be public as well, there is nonetheless a distinct private character to much of what governments do. An educated citizenry is a public good, but the public provision of education necessarily confers targeted excludable benefits on some. Teachers and their unions benefit directly from increases in education expenditures, and some contractors will benefit from the decision to build a new school, but not all contractors benefit. Government failure will be originated from the private character of the public good.

In general, wars result from the loose and fragile fabric of international organization and from non-cooperatively played international relations games, especially arms races. Arms race resembles prisoner's dilemma. In the prototypical arms race between two nations, both nations presumably prefer a minimal defense with the bulk of their resources being devoted to consumer goods and domestic public goods. Each nation, however, also prefers some small advantage over the other "just in case." And both nations fear most the other nation's potential expansion of military capabilities without a corresponding response. The result is that both nations escalate their arms stockpiles at the expense of domestic expenditures.

Olson (1968) applied lessons from this dilemma at a more basic level, asking how and why people form and maintain interest groups. Olson observes that if the purpose of a group, such as a manufacturing association or a labor union, is to provide some collective (public) benefit to its members, then such group themselves may be trapped in a dilemma. To the extent that the good or service provided is truly public, potential members of the group will be tempted to free-ride on the contributions of others, in which case the group will never form or will soon dissolve. For instance, if the firms in an industry try to form a pressure group to lobby the legislature for the limitation of competitive foreign imports, and if such limits are secured, then all firms in that industry necessarily benefit. But if this is the case, then might not each firm's management reason that its contribution to this effort ought to be minimal, so as to avoid bad publicity, the actual costs of contributing to the pressure group's maintenance, and the expenditure of political capital that it might spend lobbying for things that benefit that firm specifically.

The form of strategic misrepresentation of preferences most familiar to political scientists is vote trading, an agreement between two (or more) legislators for mutual support, even though it requires each to vote contrary to his real preferences on some legislation. Regarding vote trading, the following theorem was proved by Thomas Schwarz (Ordeshook, 1986: 92)

(Theorem 5) [Schwartz]: If corresponds to all combinations of pass-fail over a set of bills, if preferences over are separable, and if a Condorcet winner exists, sincere voting yields that winner.

6) Any game in which every player has a strictly dominant strategy has a unique solution, which is to play the strictly dominant strategy. When that solution is bad for the players, the phenomenon is called the prisoner's dilemma.

That is, if a legislature considers a series of bills, if preferences over the bills are separable, and if a Condorcet winner exists, then sincere voting yields that winner.[7] But suppose that a Condorcet winner does not exist. What outcomes are likely to prevail in that event? We cannot answer this question fully because an answer requires an analysis of coordination and coalition formation within legislature. Concerning this question, Riker and Brams (1973) suggest that if complete coordination and cooperation is not possible, then a form of the prisoner's dilemma can arise, so that vote trading yields Pareto-inefficient outcomes.

The logic of prisoner's dilemma is used to model political participation: why do people vote? Or more generally, why do they participate anyway? The source of such question is the observation that political outcomes are like public goods. Everyone must live with the policies of a president, whether one voted for him or not, or even whether one voted at all. Thus, the benefits and costs are public. But voting is not costless. Although these costs, which include the time it takes to go to the polls as well as the costs of becoming sufficiently informed to vote "correctly," are small, the benefits of voting instead of abstaining appear minuscule. Regarding this problem, there is a hypothesis that a private benefit to voting accompanies and often outweighs the private cost. Referred to as the "sense of citizen duty," such a term recognizes that much of our political socialization leads people to value the symbolic nature of political acts at least as much as their instrumental impact.

Let's go some farther with regard to election. If a majority of legislators seek benefits for their respective constituents, then all such legislators are successful at securing those benefits. In this case the resulting game is that constituents are in a prisoner's dilemma. That is, although individually they might prefer efficiency in government, it is irrational for any of them unilaterally to elect representatives who will not work for constituency-interest legislation if most others are working for that kind of legislation. This example suggests, then, that the reelection chances of an incumbent legislator depend less on the general efficiency of legislative action than on a legislator's ability to ensure that his constituents get their fair share of enactments. Further, if constituents express their dissatisfaction with the overall performance of government through their choice of, say, the president, then the electoral imperatives of the president and of Congress differ, and we should anticipate some conflict between these two branches of government, even though the same constituency elects both branches. Thus, this interpretation of a prisoner's dilemma leads to several hypotheses about legislative incentives and executive-legislative conflict.

Concerning the application of non-cooperative games, there is one more point to be noted. Political science concerns itself with the actions and decisions that people take within specific institutions. Because these institutions are human creations, they serve a purpose that reflects human design or evolutionary durability. These aims may involve inducing people to choose specific actions or leading people not to choose actions that might occur under alternative arrangements. Institutions tend to survive because they satisfy some prevailing objectives, and they may evolve or disappear if their damage to human welfare becomes patent. Hence, to understand why one institution survives while another disappears, we must try to understand the outcomes that one institution produces but another does not. That is, to understand why people choose certain political institutions instead of

7) A Condorcet winner can be defined as follows (Ordeshook, 1986: 76): Condorcet winner: If O is the set of alternative outcomes, then $x \in O$ is a Condorcet winner if, for all other $y \in O$, the number of people who strictly prefer x to y exceeds the number who strictly prefer y to x, no motion can defeat a Condorcet winner in a majority vote.

others, and to understand how these institutions might affect people's actions, we must analyze how institutions affect choice. With this in view, Ordeshook uses n-person non-cooperative voting game to model people's behavior that occur within certain institutions, and to understand the implications of certain institutions (1986: 243~301). He concludes that institutions are anything but black boxes into which we simply can plug preferences and calculate outcomes in some straightforward and mechanical way.

CONCLUSION

So far, we discussed the limitations and usefulness or relevance of non-cooperative games. Then how realistic are non-cooperative situations realistic? Participants in an n-person game may, in its course, form constant or provisional coalitions.. The assumption that people reveal their strategies simultaneously and cannot communicate their choices beforehand is a useful theoretical abstraction. It is probably rare, though, that communication among players, however imperfect, remains impossible. In response to the possibility of communication, instead of non-cooperative game theory, cooperative game theory models those situations in which communication not only is possible, but also stands as a central feature of human interaction. Communication, however, admits of another profoundly new phenomenon, the formation of coalitions, but, even in this context, communication and coordination add nothing new to game theory. For example, we could decide that coordinated action merely arguments the strategies available to people, thereby complicating both the extensive and normal form representations of situations.

As is mentioned already, nonzero-sum games are much like their zero-sum counterparts. As a consequence, games with unique Nash equilibria receive special attention, and of these games the two-person prisoners' dilemma is perhaps the most important and illuminating. Because we can represent markets with public goods as prisoners' dilemmas, and because people often call upon the government to correct market failure, the prisoners' dilemma stands at the interface between economic and political science. But the prisoners' dilemma illustrates far more than market failures. It also augments our understanding of why governments may be inefficient, why interest groups may have difficulty forming or maintaining themselves, why industries sometimes support the regulatory agencies that regulate them, why people might not vote, why nations war, and why people strategically misrepresent their evaluations of the goods and services that government provide. The prisoners' dilemma, however, is only one of the varieties of nonzero-sum games. Therefore, there is much room for the application of the non-cooperative games to study the public sphere.

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